parallel another error is introduced which will increase with frequency.

The response of the detector used in the attenuator measurements followed a square law. This was determined by use of a variable attenuator. This attenuator had been carefully calibrated against a Golay cell detector by the substitution method. The theoretical results and the measured points for one polarization in the 140 Gc and 210 Gc regions are shown in Figs. 4 and 5. The theoretical results were obtained from (1) and (2) using 2.52 as the dielectric constant of polystyrene[2] and 45° as the value for θ. The measured points of the transmitted and reflected wave for parallel polarization were within 1 db of the theoretical values. The difference can be attributed to the following: the lack of matched surfaces, surface tolerances on the coupling surfaces, and diffraction effects.

In conclusion, we can say that it is possible to build a double-prism attenuator for millimeter wave applications with good performance. Such a device can also be used as a variable coupler. If greater precision is desired in the attenuator, the outer surfaces of the prisms should be matched into free space; tolerances of less than ± 0.0015 inch must be kept and the effects of diffraction should be included.

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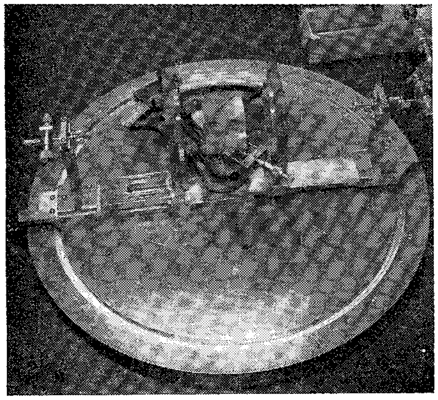

Fig. 3.—Photograph of double-prism attenuator.

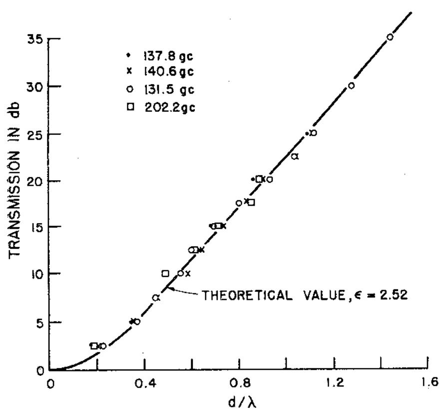

Fig. 4.—Transmission as a function of d/λ for E parallel.

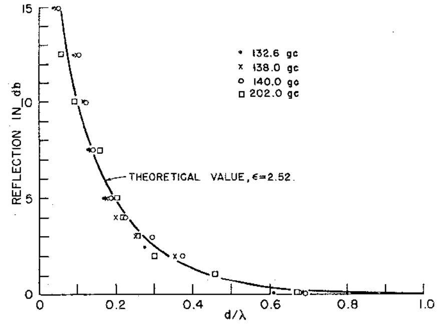

Fig. 5.—Reflection as a function of d/λ for E parallel.

[2] J. C. Wiltse, *et al.*, "Quasi-Optical Components and Surface Waveguides for the 100 to 300 Kmc Frequency Range," Electronic Communications, Inc., Timonium, Md., Sci. Rept. No. 2; November, 1960.

Application of Sampling Theorem to the Synthesis of Transmission Line Tapers and Antenna Radiation Patterns*

In this note it is demonstrated how the sampling theorem may be used to reduce the problems of transmission line taper synthesis and antenna aperture field synthesis to that of optimizing a polynomial. In both cases the problem may be stated mathematically in the following form: find the function $g(x)$ which is zero outside the domain $-\pi \leq x \leq \pi$ that will yield a desired $f(u)$ where $f(u)$ is given by the Fourier transform of $g(x)$, *i.e.*,

$$f(u) = \int_{-\pi}^{\pi} g(x) e^{ixu} dx. \tag{1}$$

In the transmission line taper problem $g(x)$ is proportional to the derivative of the logarithm of the taper impedance and $f(u)$ is proportional to the reflection coefficient, as a function of frequency, at the taper input. It is assumed that the taper impedance varies slowly enough so that the square of the reflection coefficient can be neglected.[1,2] In the antenna problem $g(x)$ is proportional to the aperture field for the one dimensional case and $f(u)$ is proportional to the radiation pattern.[3]

Let $g(x)$ be expressed as the Fourier series

$$g(x) = \sum_{n=-\infty}^{\infty} c_n e^{jnx} \tag{2}$$

* Received April 25, 1962.

[1] F. Bolinder "Fourier transforms in the theory of inhomogenous transmission lines," Proc. IRE (*Correspondence*, vol. 38, p. 1354; November, 1950.

[2] R. E. Collin, "The optimum transmission line matching section," Proc. IRE, vol. 44, pp. 539–549; April, 1956.

[3] T. T. Taylor, "Design of line-source antennas for narrow beamwidth and low side lobes," IRE Trans. on Antennas and Propagation, vol. AP-3, pp. 16–28; January, 1955.

From (1) it is found that

$$f(u) = 2\pi \sum_{n=-\infty}^{\infty} c_n \frac{\sin \pi(u-n)}{\pi(u-n)} \tag{3a}$$

$$f(n) = 2\pi c_n \tag{3b}$$

which is the sampling theorem. An arbitrary $f(u)$ would, in general, require a $g(x)$ which is nonzero over $-\infty < x < \infty$. If, however, a set of coefficients was determined by (3b) then the resultant $f(u)$ generated by this $g(x)$ which is zero outside the domain $-\pi \leq x \leq \pi$ would be equal to the arbitrarily specified $f(u)$ at the sampling points and deviate by some finite amount in between. The practical problem is thus seen to be the one specifying an $f(u)$ that can be produced by a $g(x)$ which is zero outside the domain $-\pi \leq x \leq \pi$. This may be done as follows.

Eq. (3a) may be written as

$$f(u) = 2\pi \sum_{n=-\infty}^{\infty} c_n \frac{(-1)^n \sin \pi u}{\pi(u-n)} = 2\pi \frac{\sin \pi u}{\pi u} \sum_{n=-\infty}^{\infty} c_n (-1)^n \frac{u}{u-n} \cdot \tag{4}$$

In most practical cases it is desirable to limit the expansion of $g(x)$ to a finite number of terms, say $2N+1$ terms. From (3b) it is seen that this may be accomplished by specifying $f(u)$ to have zeroes at all but $2N+1$ integer values of u. For convenience it will be assumed here that $f(u)=0$ for $|u| = n > N$. Then (4) becomes

$$f(u) = 2\pi \frac{\sin \pi u}{\pi u} \sum_{n=-N}^{N} c_n (-1)^n \frac{u}{u-n} \cdot \tag{5}$$

This expression is recognized as the partial fraction expansion of some function of the form

$$P(u) \Big/ \prod_{n=1}^{N} (u^2 - n^2)$$

where $P(u)$ is an arbitrary polynomial of degree $2N$ in u, *i.e.*,

$$\begin{aligned} \frac{P(u)}{\prod_{n=1}^{N}(u^2-n^2)} &= \sum_{m=-N}^{N}{}' \frac{P(m)}{(u-m)2m \prod_{\substack{n=1 \\ n\neq m}}^{N} (m^2-n^2)} + \lim_{u\to\infty} \frac{P(u)}{u^{2N}} \\ &= \frac{uP(u)}{u \prod_{n=1}^{N}(u^2-n^2)} \\ &= \sum_{m=-N}^{N}{}' \frac{uP(m)}{(u-m)2m^2 \prod_{\substack{n=1 \\ n\neq m}}^{n} (m^2-n^2)} + \frac{P(0)}{\prod_{n=1}^{N}(-n^2)} \end{aligned} \tag{6}$$

where the prime means omission of the term $m=0$. Thus (5) becomes

$$f(u) = 2\pi \frac{\sin \pi u}{\pi u} \frac{P(u)}{\prod_{n=1}^{N}(u^2-n^2)} \cdot \tag{7}$$

Any $f(u)$ of this form can be generated by a $g(x)$ which is zero outside the finite domain

$-\pi \le x \le \pi$. Eq. (7) states that for a given number $2N+1$ of Fourier terms in the expansion of $g(x)$ the function $f(u)$ may be modified from a $\sin \pi u/\pi u$ form by rearranging $2N$ of the zeroes of this latter function in an arbitrary manner. The synthesis procedure is now reduced to that of choosing a suitable polynomial $P(u)$. The particular case of approximating a Chebyshev behavior for $f(u)$ is discussed by Collin[2] and Taylor.[3] Further applications to transmission line taper synthesis will be presented in a later paper. Finally, the antenna optimization problem is discussed in greater detail elsewhere.[4]

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[4] R. E. Collin and S. Rothschild, "Reactive Energy in Aperture Fields and Aperture Q," to be presented at the Copenhagen Electromagnetic Symposium, June, 1962.

A Proposed Lossless Electronic Phase Shifter*

Modern antenna array designs frequently require the use of high-speed, electronically controlled phase shifters. The use of low-noise receivers now becoming available also dictates the requirement that such phase shifters have zero, or very small, losses. The use of parametric devices as active elements in such phase shifters is particularly attractive because of their high degree of reliability, high frequency capability, and excellent noise performance. This memorandum proposes such an active electronic phase shifter, which accomplishes the additional function of providing amplification or attenuation independently of its phase shift.

Although most parametric amplifiers have been designed to amplify an incoming signal directly without frequency conversion, it is possible to design low-noise parametric up-converters with the output frequency higher than the input frequency. The reverse procedure of down-converting is not practical because the power gain is always less than unity. There are two distinct modes of operation for the parametric up-converter. The output frequency may be either the sum or the difference of the pump and signal frequencies. When the lower idler is utilized ($\omega_p = \omega_i + \omega_s$), the amplifier is a negative resistance device with unlimited gain and is potentially unstable. However, when the upper idler is employed ($\omega_i = \omega_p + \omega_s$), the up-converter is stable but the gain is limited to the ratio of the idler frequency and signal frequency.

Changes in pump frequency appear as changes in the output frequency unless the idler is down-converted in a conventional crystal mixer which uses the same pump source for the local oscillator. In this case the original input frequency is recovered. Although the output frequency of the parametric up-converter/down-converter combination is independent of pump frequency the phase shift through the combination is a function of pump frequency.

It has been shown that the expression relating the phase terms in the expression for the voltages at the three different frequencies is similar to the expression relating the frequencies with an additional phase shift of $-\pi/2$ radians due to the capacitive reactance of the varactor.[1] The expression for the phase shift in a crystal mixer does not contain the term $-\pi/2$. The phase shift in a lower sideband up-converter is given by

$$\phi_p = \phi_s + \phi_i - \frac{\pi}{2}$$

where $\omega_p = \omega_i + \omega_c$, while the phase shift in a crystal mixer is given by

$$\phi_p = \phi_s + \phi_i.$$

With the aid of Fig. 1 the following set of equations may be written:

$$\phi_{s_0} = \phi_{p_2} - \phi_{i_2}$$

$$\phi_{p_2} = -\frac{2\pi l_2}{\lambda_p}$$

$$\phi_{i_2} = \phi_{i_1} - \frac{2\pi l_3}{\lambda_i}$$

$$\phi_{i_1} = \phi_{p_1} - \phi_{s_i} - \frac{\pi}{2}$$

$$\phi_{p_1} = -\frac{2\pi l_1}{\lambda_p}.$$

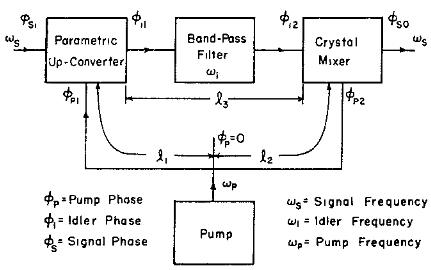

Fig. 1—Block diagram of parametric up-converter/down-converter system.

The solution for the total phase shift is

$$\phi = \phi_{s_0} - \phi_{s_i} = \frac{2\pi}{\lambda_p}(l_1 - l_2) + \frac{\pi}{2} + \frac{2\pi l_3}{\lambda_i}.$$

By similar manipulation, the phase shift of the upper sideband up-converter/down-converter combination is

$$\phi = \phi_{s_0} - \phi_{s_i} = \frac{2\pi}{\lambda_p}(l_2 - l_1) - \frac{\pi}{2} - \frac{2\pi l_3}{\lambda_i},$$

which is the negative of the previous expression.

Thus, for either the lower sideband or the upper sideband up-converter/down-converter combination, the total phase shift at a given signal frequency is a function only of the pump frequency for fixed line lengths. The choice between an upper- or lower-sideband up-converter will be affected by gain and stability requirements.

Since the rate of change of phase shift vs pump frequency is controlled by the line lengths, it is possible to design for any specified range of phase shift, 360° for example, over a fixed range of the tunable pump source. This tuning range is determined by the bandwidth of the band-pass filter, which in turn is dictated by the signal frequency.

For example, assuming a signal frequency of 200 Mc and an X-band pump, a 200-Mc band-pass filter centered on 9800 Mc would support the lower idler frequency for pump frequencies between 9900 Mc and 10,100 Mc, while suppressing the upper idler. With this set of conditions, a difference of electrical line lengths between l_2 and l_1+l_3 of 1.5 meters would be required to give 360° phase shift over the operating range.

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[1] D. B. Anderson, and J. C. Aukland, "Transmission Phase Relations of Four-Frequency Parametric Devices," presented at IRE Microwave Theory and Techniques Nat'l Symp., Washington, D. C.; May 15–17, 1961.

Oversize Waveguide Directional Coupler*

Oversize rectangular waveguide using TE_{10} mode propagation has been suggested as a means of reducing the attenuation and fabrication difficulties of transmission lines and components for frequencies above 40 Gc.[1,2] Experiments on straight sections of such transmission lines show that it is superior to pure optical transmission for short distances.[3]

A directional coupler using oversize rectangular waveguide has been constructed; experimental results are described in this communication. The initial device was de-

* Received May 1, 1962.

[1] A. F. Harvey, "Optical techniques at microwave frequencies," *Proc. IEE*, vol. 106, pt. C, pp. 141–157; March, 1959.

[2] L. Lewin, "A Note on Quasi-Optical Methods at Millimeter Wavelengths," *PBI-MRI Symp. on Millimeter Waves*, Interscience Publishers, New York, N. Y., p. 469; 1959.

[3] R. H. Garnham, "Optical and Quasi-Optical Transmission Techniques and Component Systems for Millimeter Wavelengths," Royal Radar Establishment, Malverne, England, RRE Rept. No. 3020; March, 1958.